Ceramikalc
Structural Reliability in Weibull format

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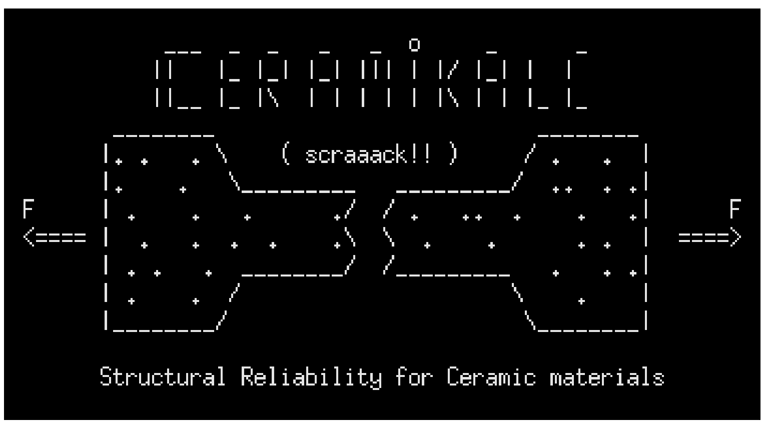

fall, 2015

1 Introduction

Ceramikalc is a simple C++ program that computes the failure probability of a fragile material under stress and outputs a VTK file for some colored plots. The failure probability is based on the Weibull distribution (see section 4).

i) perform a FEM analysis and write an input file for Ceramikalc, containing elements coordinates and stresses (see section 3)

ii) run Ceramikalc:

   - insert the parameters for the failure probability distribution
   - import the input file
   - run analysis
   - export results

iii) post-process VTK exported results with a tool such as ParaView.

   Ceramicalk currently supports only quadrilateral shell elements (such as CQUAD4), but will soon (how much soon??) support brick elements and hopefully some more stuff.

Contents

2 Compiling

Compiling ceramikalc is straightforward: get into the folder and run `make`.

3 Input file

The input file is pretty easy: ceramikalc computes the volume of every finite element and then computes the failure probability using the stress on that element. Here is the format for different finite elements.

3.1 shells (quadrilaterals)

```
x1 y1 z1 x2 y2 z2 x3 y3 z3 x4 y4 z4 stress
```

`x1,... z4` are the nodes coordinates in mm and `stress` is the stress value for the element in MPa. Values must be separated by a single space (more spaces may work, not sure about that..)

Note that the node ordering is important and must not be casual! Run ceramikalc and take a look at the "input file info" section of the menu for some more info.

Where do I insert the thickness of a shell The shell thickness (that ceramikalc needs to compute the volume) is not inserted via input file, but while running ceramikalc and importing the input file. You are going to be explicitly asked about it.

3.2 bricks (hexahedra)

```
x1 y1 z1 x2 y2 z2 x3 y3 z3 .... x8 y8 z8 stress
```

Dimensions in mm and stress in MPa. See *shells* section for more details.

4 Materials and Weibull distribution

The strength of fragile materials is dominated by imperfections in the lattice. The nature of imperfections is clearly statistical, hence the necessity for a probabilistic description. A common probability distribution for ceramic materials is the Weibull format, which is parametrized by the factors:

- k: describing the shape of the distribution

- σ_0: stress whose value corresponds to a failure probability of 63.2% (typically coming from experimentation)

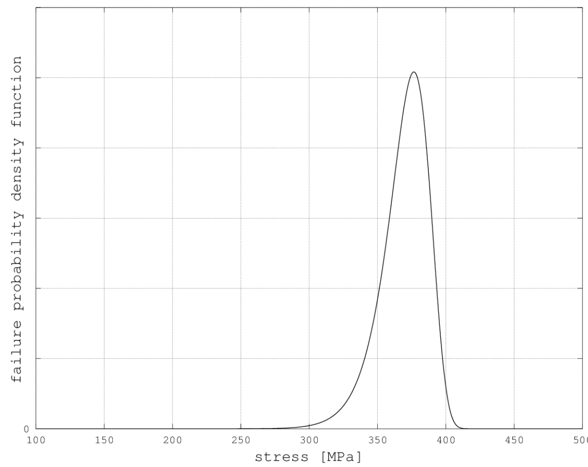

Figure 1: *Weibull distribution with parameters* $k = 26$ *and* $\sigma_0 = 377$

Let's say we have a component under stress σ: the probability that the material's weibull-described resistance is below sigma is

$$P_f = 1 - exp[-(\sigma/\sigma_0)^k] \quad (1)$$

Now, if we discretized our component via the Finite Element Method, each fine element has a certain volume and hence a certain failure probability. The reliability of the *i-th* element is:

$$R_i = exp[-V_i/V_0(\sigma_i/\sigma_0)^k] \quad (2)$$

where:

- V_i is the volume of the *i-th* element
- σ_i is the stress
- σ_0 see above
- V_0 is the reference volume, for whom the Weibull parameters such as k and σ_0 have been obtained.

The reliability of the whole piece is simply the product of the reliabilities:

$$R_{tot} = \prod_i R_i = \prod_i (1 - P_{f,i}) = 1 - P_{f,tot} \tag{3}$$

A couple results on a structured mesh:

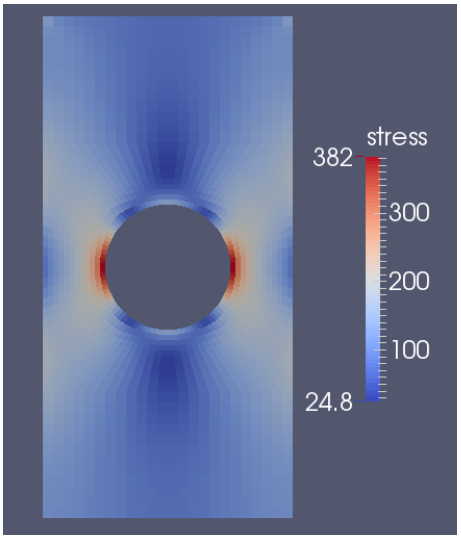

Figure 2: *Imported stress* $[MPa]$*, max principal stress*

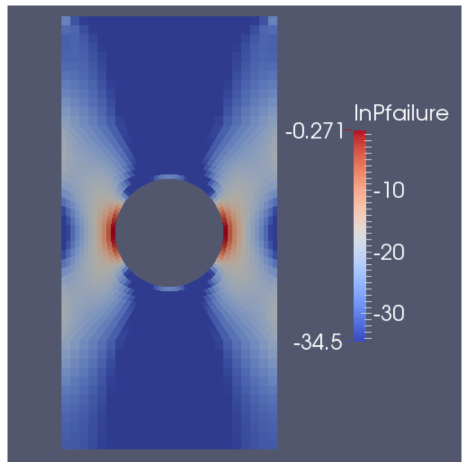

Figure 3: *Failure probability (in log scale)*

5 VTK exporting

Just a brief introduction to VTK files. there are 2 ways to create a VTK file: the *legacy* way and the *XML* format. Legacy is simpler, but does its job. Ceramikalc uses this format and imports the grid as unstructured: that means for every element, one must specify vertices and a value for the field (such as stress or probability). The stress of an element (and probability and so on) is written as a scalar value for each node.

The output file is named "`output.vtk`". Note that if another "output.vtk" file is present, it's being overwritten.

Ceramikalc exports the following values:

- stress
- Failure Probability

- natural logarithm for failure probability
- Reliability
- elements volumes

(Hint: plot reliability in logarithms under ParaView!)

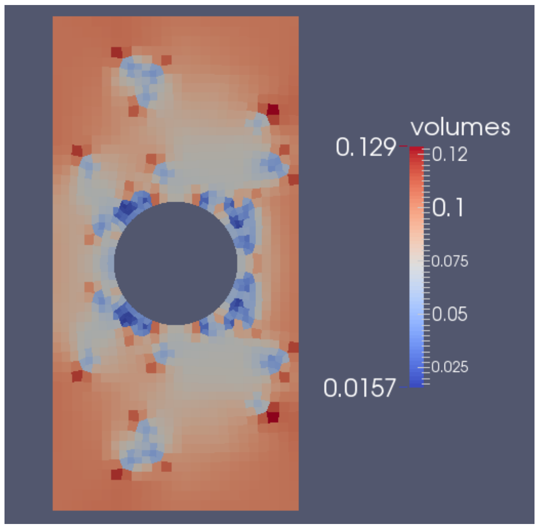

Figure 4: *Volume values for a not very uniform mesh*

6 References

Affidabilità delle costruzioni meccaniche - Stefano Beretta - Springer, 2009